INSTABILITY OF THE ENTROPY LAYER ON A BLUNT PLATE IN SUPERSONIC GAS FLOW

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An entropy layer is formed on the surface of blunt bodies in supersonic gas flow [1]. Under the entropy layer there is a boundary layer, which grows in the downstream direction. Transition from laminar to turbulent flow results from the development of unstable perturbations in the boundary layer [2, 3]. Experiments on cones and flat plates have shown [4, 5] that if one increases the blunting of the leading edge, the extent of the laminar flow section increases, reaches a maximum, and then decreases. Belolipetskii and Stepanova [6] were able to explain in part the displacement downstream of the transition zone as being due to a decrease of local Reynolds number, computed from the flow parameters at the bottom of the entropy layer, but this model does not describe the reversal of transition.

The phenomenon of reversal may arise from the following causes. The entropy layer deforms the mean flow profile in the laminar boundary layer, causing the latter to become unstable. Using the asymptotic theory of free interaction reference [7] determined that the entropy effect leads to instability of the boundary layer to small perturbations. It was shown [8, 9] from numerical computations that the entropy layer on a blunt flat plate acts in a destabilizing way on the second unstable boundary layer mode. The other possibility is instability of the entropy layer itself. Self oscillations developing in the entropy layer are free to penetrate into the boundary layer and thereby initiate early transition to turbulence. Khan et al. [8, 9] investigated numerically the stability characteristics of the entropy layer on a blunt flat plate. However, in formulating the problem an inaccuracy occurred in the boundary conditions for perturbations at the bottom of the entropy layer.

This paper uses linear theory to analyze self oscillations in the entropy layer. The method of matched expansions is used to obtain the correct boundary conditions for the perturbations. It is shown that in the leading approximations the entropy layer instability is described by inviscid equations. The author performs a numerical and asymptotic analysis of acoustic self oscillations and of the unstable mode.

1. We consider flow of a supersonic gas over a blunt flat plate. At distance L from the leading edge a boundary layer is formed of thickness $\delta_e = L/\sqrt{R_L}$ and an entropy layer of thickness on the order of the blunting radius r_N. The Reynolds number is $R_L = LU_e\rho_e/\mu_e$, based on the flow parameters at the outer edge of the boundary layer. We investigate the flow region where $\delta_e \ll r_N \ll L$, i.e., the boundary layer lies at the bottom of the entropy layer. Since $r_N \ll L$, the density discontinuity degenerates into a Mach wave and the flow parameters at the outer edge of the entropy layer are close to those of the oncoming stream U_∞, ρ_∞, μ_∞.

We make nondimensional the longitudinal coordinate x and the transverse coordinate y with respect to the blunting radius r_N; and the density ρ, the x and y velocity components U and V, the temperature T, the pressure P and the viscosity μ with respect to their values in the oncoming stream. As the characteristic boundary layer thickness we take the scale $\delta = L/\sqrt{LU_\infty\rho_\infty/\mu_\infty} \sim \delta_e$. We introduce the small parameters $\varepsilon = \delta/r_N$ and $\varepsilon_0 = r_N/L$. The structure of the mean flow is shown in Fig. 1. It follows from the asymptotic analysis of [10] that in the inviscid regions 2 and 3 with characteristic scale r_N the basic flow has the form

$$P = 1 + O(\varepsilon_0^2\varepsilon),\quad T = T_2(y) + O(\varepsilon_0\varepsilon) + O(\varepsilon_0^2),$$
$$U = U_2(y) + O(\varepsilon_0\varepsilon) + O(\varepsilon_0^2),\quad V = O(\varepsilon_0\varepsilon) + O(\varepsilon_0^2); \tag{1.1}$$

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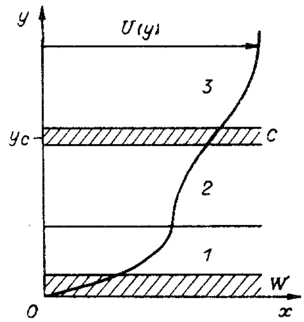

Fig. 1

$$T_2(y) = \left[1 - \frac{2(M^2-1)^2}{(\gamma+1)M^2(M^2-1+\psi^2)}\right] \times$$
$$\times \left[1 + \frac{2\gamma(M^2-1)^2}{(\gamma+1)M^2}\left(\frac{M^2}{M^2-1-M^2\psi^2}\right)\right]^{1/\gamma},$$
$$U_2(y) = \left[1 - \frac{2(T_2-1)}{(\gamma-1)M^2}\right]^{1/2}, \quad y = \int_0^\psi \frac{T_2}{U_2}d\psi \tag{1.2}$$

(M is the Mach number of the oncoming stream, γ is the adiabatic index, and ψ is the stream function).

In the boundary layer (region 1 of Fig. 1) the relations [9] are valid

$$P = 1 + O(\varepsilon_0^2\varepsilon^2), \quad U = U_1(y_1) + O(\varepsilon_0\varepsilon) + O(\varepsilon U_2'(0)),$$
$$T = T_1(y_1) + O(\varepsilon_0\varepsilon) + O(\varepsilon U_2'(0)), \tag{1.3}$$

where $y_1 = y/\varepsilon$ is the inner variable; the primes denote differentiation with respect to y; U_1 and T_1 are the profiles of velocity and temperature in the plane-parallel zero-gradient boundary layer, which as $y_1 \to \infty$ tend to the limiting values $U_1 \to U_2(0) = U_e$, $T_1 \to T_2(0) = T_e$.

On the basic flow we superimpose the two-dimensional perturbation

$$(u, v, p, \theta_0, \rho_0) = (f, \alpha\varphi, \pi, \theta, r) \exp(i\alpha x - i\omega t).$$

Here u, v, p, θ_0, ρ_0 are pertubations of the longitudinal and vertical velocity, pressure, temperature and density; α is the wave number; ω is the frequency, referenced to U_∞/r_N; and t is dimensionless time.

We consider perturbations of characteristic wavelength $\lambda \sim r_N$ and frequency $\sim U_\infty/r_N$, i.e., $\alpha = O(1)$, $\omega = O(1)$, and the phase velocity is $c = \omega/\alpha = O(1)$. We consider that $R = r_N\rho_\infty U_\infty/\mu_\infty \gg 1$, and for the analysis we use the Dana–Lin system [2]

$$\rho[i(U-c)f + U'\varphi] + i\pi/(\gamma M^2) = \varepsilon_0\varepsilon^2\mu f''/\alpha,$$
$$i\alpha^2\rho(U-c)\varphi + \pi'/(\gamma M^2) = 0, \ i(U-c)r + \rho'\varphi + \rho(if + \varphi') = 0,$$
$$\rho[i(U-c)\theta + T'\varphi] + (\gamma-1)(if + \varphi') = \varepsilon_0\varepsilon^2(\mu\gamma/\sigma\alpha)\theta'',$$
$$\pi/P = r/\rho + \theta/T. \tag{1.4}$$

In writing this system we make the assumption that $R^{-1} = \varepsilon_0\varepsilon^2$; and σ is the Prandtl number. On the perturbation we impose the boundary conditions

$$\varphi(0) = f(0) = \theta(0) = 0, \ (\varphi, f, \theta, \pi) \to 0, \ y \to \infty. \tag{1.5}$$

2. In the first and fourth equations of the system (1.4) there is a small parameter with a leading derivative. The basic flow is described by the asymptotes (1.1) and (1.3) in the regions $y = O(1)$, $y_1 = O(1)$. We shall use the method of matched expansions [11]. We consider the case when the phase velocity $c_r = \mathrm{Re}(\omega/\alpha) > U_2(0)$. Then there is a critical layer in which $U_2(y_c) = c_r$, and it is either absent ($c_r > 1$) or present in the entropy layer 2. The characteristic regions for the perturbation are shown in Fig. 1. Near the wall there forms a viscous sublayer of thickness $\delta_1 \sim R^{-1/2} = \varepsilon_0^{1/2}\varepsilon$. Since the ratio of the thickness of the viscous sublayer to that of the boundary layer is $\delta_1/\delta \sim \varepsilon_0^{1/2} \ll 1$, the vis-

cous sublayer lies at the bottom of the boundary layer. Near the point y_c there forms a critical layer of thickness $\delta_2 \sim R^{-1/3} = (\varepsilon_0 \varepsilon^2)^{1/3}$. The viscous sublayer W and the critical layer C are separated by the inviscid zones 1 and 2.

According to Eq. (1.1) for the main flow we write the perturbations in the inviscid regions 2 and 3 in the form

$$(f, \varphi, \pi, \theta, r) = (f_0, \varphi_0, \pi_0, \theta_0, r_0) + O(\varepsilon\varepsilon_0) + O(\varepsilon^2) + O(\varepsilon_0^2). \tag{2.1}$$

Substituting Eq. (2.1) into Eq. (1.4) we obtain in the first approximation the system with no viscous terms

$$\varphi_0' - \frac{U_2'}{U_2 - c}\varphi_0 = -i\frac{(U_2 - c)}{\gamma}\left[1 - \frac{T_2}{M^2(U_2 - c)^2}\right]\pi_0,$$
$$\pi_0' = -i\alpha^2\gamma M^2 \frac{(U_2 - c)}{T_2}\varphi_0. \tag{2.2}$$

To find analytical solutions of system (2.2) we must have rigid limits on the wave number α. To avoid this the system was integrated numerically. The critical point y_c moved away downstream in the complex y plane. The correct standoff was determined by matching solutions in the critical layer with solutions in regions 2 and 3 [3, 12]. The integration was performed from the outer edge of the entropy layer to the wall. At the outer edge we used the asymptote satisfying the condition for attenuation of the perturbations:

$$y \to \infty, \ (\varphi_0, \ \pi_0) \sim \exp(\lambda y), \ \lambda = \sqrt{\alpha^2 - M^2(\alpha - \omega)^2}, \ \operatorname{Re}\lambda < 0. \tag{2.3}$$

We shall construct the solution in region 1. In system (1.4) we convert to the inner variable $y_1 = y/\varepsilon$. We make the expansions

$$f = f_{00} + \varepsilon_0 f_{01} + \ldots + \varepsilon(f_{10} + \varepsilon_0 f_{11} + \ldots) + \ldots,$$
$$\varphi = \varepsilon(\varphi_{00} + \varepsilon_0\varphi_{01} + \ldots) + \varepsilon^2(\varphi_{10} + \varepsilon_0\varphi_{11} + \ldots) + \ldots,$$
$$\pi = \pi_{00} + \varepsilon^2(\pi_{01} + \varepsilon_0\pi_{02} + \ldots) + \ldots,$$
$$\theta = \theta_{00} + \varepsilon_0\theta_{01} + \ldots + \varepsilon(\theta_{10} + \varepsilon_0\theta_{11} + \ldots) + \ldots$$

Taking account of Eq. (1.3) for the main flow, in the leading order we obtain the system

$$\frac{1}{T_1}\left[i(U_1 - c)f_{00} + \frac{dU_1}{dy_1}\varphi_{00}\right] = -\frac{i\pi_{00}}{\gamma M^2}, \quad \frac{d\pi_{00}}{dy_1} = 0,$$
$$i(U_1 - c)\left(\pi_{00} - \frac{\theta_{00}}{T_1}\right) - \frac{\varphi_{00}}{T_1}\frac{dT_1}{dy_1} + if_{\infty} + \frac{d\varphi_{00}}{dy_1} = 0,$$
$$\frac{1}{T_1}\left[i(U_1 - c)\theta_{00} + \varphi_{00}\frac{dT_1}{dy_1}\right] + (\gamma - 1)\left(if_{00} + \frac{d\varphi_{00}}{dy_1}\right) = 0.$$

Its solution will be

$$\pi_{00} = B_1, \theta_{00} = \frac{\gamma - 1}{\gamma}B_1 T_1 - \frac{\varphi_{00}}{i(U_1 - c)}\frac{dT_1}{dy_1},$$
$$\varphi_{00} = (U_1 - c)\left\{B_2 - \frac{iB_1}{c}\int_0^{y_1}\left[1 - \frac{T_1}{M^2(U_1 - c)^2}\right]dy_1\right\}. \tag{2.4}$$

In region W we convert to the variable $\eta = y_1/\sqrt{\varepsilon_0}$ and make the expansions

$$(U_1, T_1, \rho_1, \mu_1) = (U_w, T_w, \rho_w, \mu_w) + \varepsilon_0^{1/2}\left(\frac{dU_1}{dy_1}, \frac{dT_1}{dy_1}, \frac{d\rho_1}{dy_1}, \frac{d\mu_1}{dy_1}\right)_w \eta + \ldots,$$
$$f = f_{00} + \varepsilon_0^{1/2}f_{01} + \ldots + \varepsilon(f_{10} + \varepsilon_0^{1/2}f_{11} + \ldots) + \ldots,$$
$$\varphi = \varepsilon\varepsilon_0^{1/2}(\varphi_{00} + \varepsilon_0^{1/2}\varphi_{01} + \ldots) + \varepsilon^2\varepsilon_0^{1/2}(\varphi_{10} + \varepsilon_0^{1/2}\varphi_{11} + \ldots) + \ldots,$$
$$\pi = \pi_{00} + \varepsilon^2\varepsilon_0(\pi_{01} + \varepsilon_0^{1/2}\pi_{02} + \ldots) + \ldots, \quad \theta = \theta_{00} + \varepsilon_0^{1/2}\theta_{01} + \ldots$$
$$\ldots + \varepsilon(\theta_{10} + \varepsilon_0^{1/2}\theta_{11} + \ldots) + \ldots$$

Here the subscript w denotes the value for $y_1 = 0$. In the leading order we have the system

$$-\frac{ic}{T_w} f_{00} = -\frac{i}{\gamma M^2}\pi_{00} + \mu_w \frac{d^2 f_{00}}{d\eta^2}, \quad \frac{d\pi_{00}}{d\eta} = 0, \quad -ic\left(\pi_{00} - \frac{\theta_{00}}{T_w}\right) + $$
$$+ if_{00} + \frac{d\varphi_{00}}{d\eta} = 0,$$
$$-ic\frac{\theta_{00}}{T_w} = -(\gamma-1)\left(if_{00} + \frac{d\varphi_{00}}{d\eta}\right) + \frac{\gamma\mu_w}{\sigma}\frac{d^2\theta_{00}}{d\eta^2}, \quad f_{00}(0) = \varphi_{00}(0) = \theta_{00}(0) = 0.$$

Its solution has the form

$$\pi_{00} = A_1, f_{00} = \frac{A_1 T_w}{c\gamma M^2}[1 - \exp(-k_1\eta)], \theta_{00} = \frac{\gamma-1}{\gamma} A_1 T_w [1 - \exp(-k_2\eta)],$$
$$\varphi_{00} = A_1\left[D_0\eta + \frac{D_1}{k_1}(1 - \exp(-k_1\eta)) + \frac{D_2}{k_2}(1 - \exp(-k_2\eta))\right], \quad k_1 = \sqrt{\frac{-ic}{\mu_w T_w}}, \tag{2.5}$$
$$\operatorname{Re} k_1 > 0, \quad k_2 = \sqrt{\sigma} k_1, \quad D_0 = \frac{ic}{\gamma}\left[1 - \frac{T_w}{c^2 M^2}\right], \quad D_1 = \frac{iT_w}{c\gamma M^2}, \quad D_2 = ic\frac{\gamma-1}{\gamma}.$$

Comparing Eqs. (2.1), (2.4), and (2.5), we find the boundary condition for Eq. (2.2)

$$\varphi_0(0) = \varepsilon(1-c)\pi_0(0)\left\{\frac{i}{\gamma}\int_0^\infty\left[\frac{T_1}{M^2(U_1-c)^2} - \frac{T_e}{M^2(U_e-c)^2}\right]dy_1 + O\left(\varepsilon_0^{1/2}\right)\right\}. \tag{2.6}$$

We note that in the case of neutral perturbations the integrand expression is the difference between the squares of the local sound speeds in the coordinate system moving with phase velocity c.

Thus, two-dimensional perturbations in the entropy layer are described by the system of inviscid equations (2.2) with boundary conditions (2.3) and (2.6). In the main approximation we use the no-penetration condition $\varphi_0(0) = 0$. Here a perturbation of the entropy layer does not affect the boundary layer. In the next approximation Eq. (2.6) contains the integral characteristic of the mean flow in the boundary layer. To describe perturbations in the entropy layer references [8, 9] used the Lees–Lin system of equations, which account for all the viscous terms of the linearized Navier–Stokes equations. Here the boundary condition (1.5) converts to the outer edge of the boundary layer with no change. It can be seen from the foregoing analysis that this procedure is incorrect. Even in the inviscid approximation the boundary condition at the bottom of the entropy layer depends on the mean flow in the boundary layer. The reason is that the viscous sublayer W lies at the bottom of the boundary layer and is separated from the entropy zone by the inviscid region 1.

The problem of Eqs. (2.2), (2.3), and (2.6) is an eigenvalue problem. The wave number α and frequency ω are connected by the dispersion relation $D(\omega, \alpha) = 0$. Analysis of stability of the entropy layer reduces to examining properties of the roots of the dispersion equations $\omega(\alpha)$ or $\alpha(\omega)$.

3. The results of Sec. 2 are also valid for short-wave perturbations satisfying the condition $1 \ll \alpha \ll \varepsilon^{-1}$, i.e., with a wavelength much exceeding the boundary layer thickness, but much less than the entropy layer thickness. The system (2.2) can be reduced to a second-order equation for the pressure perturbation. In the main approximation for ε we come to the problem

$$\pi_0'' - \left(\frac{2U_2'}{U_2 - c} - \frac{T_2'}{T_2}\right)\pi_0' + \alpha^2 q\pi_0 = 0,$$
$$\pi_0'(0) = 0, \quad \pi_0(y) \to 0, \quad y \to \infty, \quad q = M^2(U_2 - c)^2/T_2 - 1.$$

Using the VKB method [13] in an analogous way to what was done in [14] for short-wave perturbations of the boundary layer, we obtained the following results. In the entropy layer there is a discrete set of acoustic modes with phase speeds $U_2(0) + a_2(0) < c < 1 + 1/M$, where $a_2(0) = \sqrt{T_2(0)}/M$ is the sound speed at the bottom of the entropy layer. From the numerical results of Sec. 4 we obtain $U_2(0) + a_2(0) > 1$, i.e., for the given modes $c > 1$ and there is no critical layer. In this case the acoustic modes are neutral, and their eigenfunctions and the dispersion relation have the form

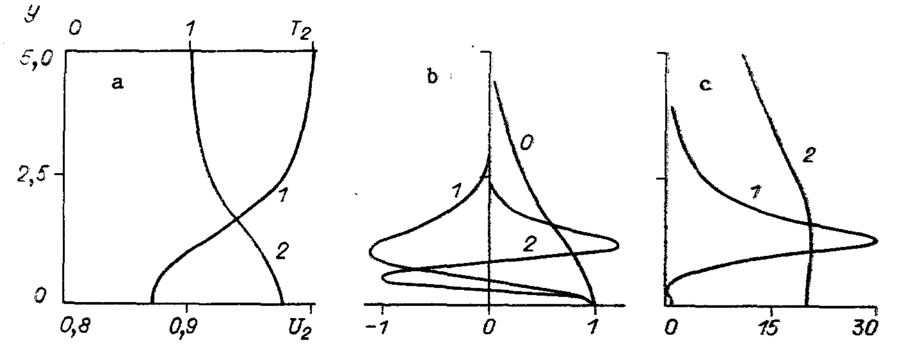

Fig. 2

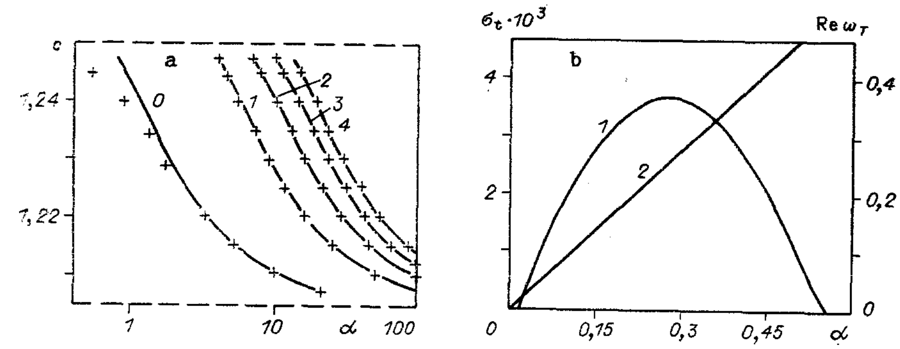

Fig. 3

$$\pi_{0n} = \frac{U_2 - c}{\sqrt{T_2}} z^{1/4} (-q)^{-1/4} \,\mathrm{Ai}\,(z),$$
$$\frac{2}{3} z^{3/2} = \alpha \int_{y_a}^{y} \sqrt{-q}\, dy, \quad \alpha \int_{0}^{y_a} \sqrt{q}\, dy = \frac{\pi}{4} + \pi n. \qquad (3.1)$$

Here Ai(z) is the Airy function [15]; y_a is the turn point at which $U(y_a) = c - a(y_a)$; and n is a large integer, the mode number. The eigenfunctions $\pi_{0n}(y)$ oscillate in the region $0 < y < y_a$ and decay exponentially for $y > y_a$. The number of zeros of the function π_{0n} is n. If n is fixed, and the phase speed $c \to U_2(0) + a_2(0)$, then the turn point $y_a \to 0$ and the wave number $\alpha_n \to \infty$. For phase speeds $c > 1 + 1/M$ the solution of the equation oscillates, with no attenuation as $y \to \infty$. The eigenvalue wave numbers fall in the region of the continuum spectrum.

4. The system of equations (2.2), (2.3), and (2.6) was integrated numerically by a fourth-order Runge–Kutta method. The main flow was computed from Eq. (1.2). The Newton method was used to search for eigenvalues. The calculations mentioned above were performed for $\varepsilon = 0$. Figure 2a shows profiles of the mean flow and the entropy layer for M = 4, and adiabatic index $\gamma = 1.4$. Curve 1 shows $U_2(y)$, and curve 2 shows $T_2(y)$. Figure 3a shows the phase speed c_n as a function of the wave number α for neutral acoustic modes. The upper broken curve shows the boundary $c = 1 + 1/M$, above which there is a continuum spectrum; the lower curve shows the limit $c = U_2(0) + a_2(0)$. The solid lines show the dispersion relation (3.1), and the number of curves is the number of modes, n. Beginning with n = 1, the asymptote agrees well with the numerical computation denoted by crosses. Figure 2b shows the eigenfunctions $\pi_n(y)$ of the acoustic modes, computed numerically for the phase speed c = 1.23 (the curve number is n). In accordance with the asymptotic model of Sec. 3 the eigenfunctions oscillate below the turn point $y_a = 1.48$ and decay exponentially for $y = y_a$.

In the spectrum there is one unstable mode whose characteristics are shown in Fig. 3b. The increment of time $\sigma_t = \mathrm{Im}\,\omega_T(\alpha)$ (curve 1) and of frequency $\mathrm{Re}\,\omega_T(\alpha)$ (curve 2) are typical for an inviscid instability of shear flows (here and below the index T denotes characteristics of an unstable mode). The eigenfunctions of the instability waves for $\alpha_T = 0.2799 - i0.00393$, $\omega = 0.2583$ are shown in Fig. 2c (curve 1 is the modulus of the x component of mass flow fluctuations, and 2 is the modulus of the pressure perturbations $|\pi_T| \cdot 20$). The mass

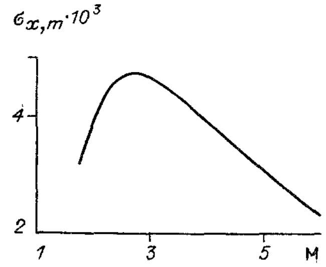

Fig. 4

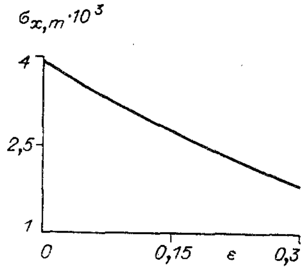

Fig. 5

flow perturbation has a clearly pronounced maximum in the critical layer, which agrees qualitatively with the experimental data obtained on a blunt cone at M = 8 [16], and on a flat plate at M = 2 [17].

To analyze the type of instability we must examine the behavior of the roots $\alpha(\omega)$ of the dispersion equation for complex ω, lying in the band $0 < \mathrm{Im}\,\omega \leq \Omega$, where $\Omega = \max[\mathrm{Im}\,\omega_T(\alpha)]$. Numerical computations have shown that the dispersion curve $\alpha_T(\omega)$, corresponding to an unstable mode, does not have a branch point in the given band. Then, according to the criteria set out in [18, 19], the instability is convective. That is, a perturbation excited locally in x and t by an external action, attenuates with time at the fixed point x. As $t \to \infty$ the perturbation is converted to a wave packet of instability. The "bulge" of the packet is transported with the group speed $V = d\,\mathrm{Re}\,\omega_T(\alpha_s)/d\alpha$ [α_s is determined from the condition $d\,\mathrm{Im}\,\omega_T(\alpha_s)/d\alpha = 0$]. It follows from the characteristics of $\omega_T(\alpha)$ (see Fig. 3b) that $V > 0$ (the instability evolves downstream).

If the instability wave is excited by some source acting at a given frequency ω, then it grows downstream according to the spatial increment $\sigma_x = -\mathrm{Im}\,\alpha_T(\omega)$. Figure 4 shows the dependence of the maximum increments $\sigma_{x,m} = \max \sigma_x(\omega)$ on the Mach number. The greatest instability is reached for M = 2.75.

Figure 5 illustrates the influence of the boundary layer on a three-dimensional instability of the entropy layer at M = 4. The boundary layer was computed for $\sigma = 0.72$, and the stagnation temperature at the outer edge was 310 K. The wetted surface was assumed to be thermally insulated. In the computations the coordinate y_1 was nondimensionalized to the boundary layer displacement thickness δ^*, and therefore the parameter $\varepsilon = \delta^*/r_N$. The boundary layer proved to have a stabilizing influence on the unstable mode of the entropy layer. From the computations made in the range of temperature factors 0.2-1.8 it can be seen that the boundary layer on a cold wall stabilizes the unstable mode of the entropy layer more strongly than on a heated wall.

In conclusion we note that as the boundary layer thickens downstream the self-oscillations of the entropy layer become closer in scale to the boundary layer modes. In the absorption regime, for which $\delta \sim r_N$, one would expect a rapid growth of the intensity of fluctuations in the boundary layer due to its being penetrated by perturbations excited in the entropy zone. This effect was observed in the experiments of [16].

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